Alpha rhythm emerges from large-scale networks of realistically coupled multicompartmental model cortical neurons

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Abstract. Cortical pyramidal and stellate neurons were simulated using the GENESIS simulation package. Model neurons were leaky integrate-and-fire and consisted of from four to nine passive compartments. Neurophysiological measurements, based on single-cell recordings and patch-clamp experiments, provided estimations for the simulation of cortical neurons: transmitter-activated conductances, passive membrane time constants and axonal delays. Network connectivity was generated using a previously described probabilistic scheme based on known cortical histology, in which the probability of connections forming between one neuron and another fell off monotonically with increasing inter-cellular separation. Simulations of up to 6400 cortical neurons, approaching the scale of an individual cortical column, confirmed previous findings with smaller networks. Limit-cycle behaviour emerged in the network, in the frequency in the range of the mammalian alpha and beta rhythms (8–20 Hz). Contrary to expectation, near-linear relationships were found between the mean soma membrane potential and neuronal firing probability. Some of the implications for cortical information processing, in particular the dynamical interactions between the neuronal and larger scales, are discussed.

1. Introduction

Neurophysiological phenomena can be grouped into scale hierarchies, such that the properties of one scale of phenomena can be understood in terms of the properties of adjacent scales (Churchland 1986). For example, the membrane dynamics of individual neurons can be understood, on the one hand, as arising out of smaller scale transmitter and channel kinetics, and on the other hand, as taking place in the context of the rise and fall of ongoing network activity. The electroencephalogram (EEG) provides one sensitive measure of cerebral cortical activity which may be used as an experimental ‘yardstick’ for the development of models of cortical functioning. This consideration motivates the present paper. The relevance of simulating EEG is shown by the close correlation of macroscopically recorded EEG with certain cognitive events. For example, in states of high arousal the EEG approaches a state characterized as desynchronization, in which spectral power resembles 1/f noise (Freeman and van Dijk 1987). Components of event-related potentials, the post-stimulus time-locked EEG, have been correlated with specific cognitive events (Wright and Kydd 1992 and references

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cited therein). EEG recorded from patients with a variety of psychiatric disorders (bipolar affective, obsessive compulsive and paranoid schizophrenia) can be used to categorize these patients with success rates of up to 80% (Tsai et al 1993, Hazarika et al 1997). This indicates a potentially close relationship between EEG activity and mental and/or behavioural states.

The field of attractor neural network (ANN) design is the most visible attempt at reconciling individual neuron behaviour with the resultant network behaviour, despite its necessarily limited biological fidelity. When the ANN properties are compared to their biological counterparts, the point of comparison is largely single- or multi-unit recordings (Amit 1990). The consideration of EEG in modelling may cast light on the neurophysiological mechanisms of cognition at larger scales—up to the scale of the entire brain. Theoretical approaches aimed at elucidating the relationship between these scales has received a great deal of attention in recent years being best typified by the research program of synergetics (Haken 1983, 1996, Jirsa and Haken 1996) in which macroscopic phenomena are seen as partially arising out of and constraining microscopic fluctuations. While there have been some successful applications of this approach (Kelso et al 1991, 1992) in understanding the phenomenology of macroscopic electrocortical phenomena, such as the MEG (magnetoencephalogram) and EEG, such approaches largely leave unanswered the formal relationship between the microscopic and macroscopic that incorporates what is known both physiologically and anatomically about cortex.

Many important insights have been gained through the study of large networks of simplified neurons. However, most of these models are poor approximations of the dynamical complexity of real neurons and it is unlikely that underlying mechanisms, even considered as mathematical abstractions, have been adequately captured. For example, transmitter-activated channels (TAC) are often assumed to be described by a first-order process, whereas experiment suggests that a second- or higher-order process is more realistic (Freeman 1991, 1992). Such differences have important consequences for resulting model network stability. Another problem arises from the connectivity schemes used in ANNs, which are generally some variant of a fully interconnected network. This is unrealistic, for the brain in particular, and for any computing device which must be physically realized. As a result of these shortcomings, it is difficult to make any comparisons between the properties of most existing ANN methodologies and empirical measurements from real brains.

Our goal in this paper was to correct the most accessible of these shortcomings, so as to determine whether properties present in real neuronal networks—and especially those properties observable in the brain’s gross electrical fields—then emerge.

1.1. Dynamical theories of EEG

Many theories of the mammalian EEG adopt what can be called a mass action approach (Freeman 1975, 1992) in which locally connected ensembles of neurons are reduced to lumped aggregates. The most common form of aggregate or macro-unit used in such modelling corresponds approximately to the KII set of Freeman (1975). KII sets are lumped approximations of interactive (not necessarily exclusively axo-synaptic) aggregates of inhibitory and excitatory cells—which include all combinations of feedforward and feedback relationships between the functionally distinct neural populations. However, most models of electro-rhythmogenesis attempt an explanation from a time and/or space coarse-grained perspective in which much of the random cellular behaviour is smoothed out for the purposes of computational simplicity and analytical tractability (Freeman 1975, 1992, Nunez 1981, 1989, van Rotterdam et al 1982, Liley 1997, Robinson et al 1997). This has resulted in questions of cellular connection asymmetry and detailed dynamics being de-emphasized in
2. Compartmental models of neurons

The model neurons described in this paper incorporate as much neurophysiological realism as is computationally tractable. Neuron dendrite and soma were modelled as multiple connected compartments in which the electrical behaviour was described by the following first-order differential equation (Wilson and Bower 1989):

\[
\frac{dV_{m,j}(t)}{dt} = \sum_{k=1}^{n_{\text{compartments}}} \left[ E_k - V_{m,j}(t) \right] g_{jk}(t) + \frac{E_m - V_{m,j}(t)}{r_{m,j}} + I_{a(j)} + I_{\text{inject}(j)}
\]

where \( V_{m,j} \) is the transmembrane potential for the \( j \)th compartment. \( E_m \) is the resting membrane potential and is assumed to be equal for all compartments. \( g_{jk}(t) \) is the total membrane conductance for the \( k \)th ionic species in the \( j \)th compartment at time \( t \). \( E_k \) is the equilibrium potential for the \( k \)th ionic species. \( I_{a(j)} \) is the total axial current entering the compartment \( j \) from adjacent connected compartments, and \( I_{\text{inject}(j)} \) is any explicit current injection into compartment \( j \). \( c_{m(j)} \) and \( r_{m(j)} \) are the total membrane capacitance and total membrane leakage resistance, respectively.

Each neuron had up to nine compartments (figure 1). With respect to the effects of attenuation of the synaptic input with distance from the soma, models of neurons containing as few as nine compartments have been shown to reproduce accurately the soma membrane
potential due to distal synaptic input; there is little loss of accuracy compared to simulating 50 or 60 compartments (Bush and Sejnowski 1993). Table 1 shows the dimensions of the reduced models of typical pyramidal neurons, the figures being from the cat visual cortex. Stellate cells are modelled by including only the basal geometry. Axonal action potential generation in model neurons was described by a stereotypical process in which an action potential (spike) was generated when the soma membrane potential exceeded a threshold, conditional on the neuron not having fired within a previous time period equal to the absolute refractory period. Each neuron in the network had a threshold drawn from a Gaussian distribution (for further details, see Liley 1995). TAC kinetics were modelled as second-order linear differential systems driven by input spikes arising from connected neurons. The input spikes were modelled as Dirac delta functions. Both excitatory and inhibitory synaptic actions were characterized by a

<table>
<thead>
<tr>
<th>Table 1. The dimensions of reduced models of cat visual cortex pyramidal cells. Note the similarity of the basal component for each model. See figure 1 for the topology of each neuron model. A model level V pyramid was used in all simulations. Table adapted from Bush and Sejnowski (1993).</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Layer V pyramid</strong></td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>(µm)</td>
</tr>
<tr>
<td>Soma</td>
</tr>
<tr>
<td>Apical trunk</td>
</tr>
<tr>
<td>Obliques</td>
</tr>
<tr>
<td>Apical I</td>
</tr>
<tr>
<td>Apical II</td>
</tr>
<tr>
<td>Apical tuft</td>
</tr>
<tr>
<td>Basal trunk</td>
</tr>
<tr>
<td>Basal I</td>
</tr>
<tr>
<td>Basal II</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Model parameters for homotypical cortex.</th>
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</thead>
<tbody>
<tr>
<td><strong>Symbol</strong></td>
</tr>
<tr>
<td>$R_m$</td>
</tr>
<tr>
<td>$C_m$</td>
</tr>
<tr>
<td>$R_a$</td>
</tr>
<tr>
<td>$R_e$</td>
</tr>
<tr>
<td>$\tau_1$</td>
</tr>
<tr>
<td>$\tau_2$</td>
</tr>
<tr>
<td>$E_m$</td>
</tr>
<tr>
<td>$E_e$</td>
</tr>
<tr>
<td>$E_i$</td>
</tr>
<tr>
<td>$G_{\text{max}}$ (excitatory)</td>
</tr>
<tr>
<td>$G_{\text{max}}$ (inhibitory)</td>
</tr>
<tr>
<td>$t_{\text{abs}}$</td>
</tr>
<tr>
<td>$t_r$</td>
</tr>
<tr>
<td>$\langle \theta \rangle$</td>
</tr>
<tr>
<td>$\langle \theta^2 \rangle$</td>
</tr>
<tr>
<td>$t_e$</td>
</tr>
<tr>
<td>$\theta_0$</td>
</tr>
</tbody>
</table>
relatively rapid time to peak. The values chosen for transmitter kinetic dynamics correspond approximately to ‘fast’ excitatory (AMPA/kainate kinetics) and ‘fast’ inhibitory (GABA_A kinetics).

In the set of simulations described herein, synaptic weights were not dynamically modified with ongoing network activity. The various parameters used in the simulations are shown in table 2.

3. Estimating cortical connectivity

The method for estimating cortical connectivity has been described elsewhere (Liley and Wright 1994) and will be only briefly summarized here. The estimates of cortical connectivity presented here apply only to the short-range intra-cortical connections by which model neurons in the network were connected. In essence, this method relies on the physical limitations to connectivity within the space of the cortical sheet, given certain assumptions about the geometry of dendritic and axonal trees. The basic assumption is that if an axonal segment comes into contact with a dendritic segment, then a synapse will form. Other assumptions of this method are:

- The basal dendritic system of a neuron has an approximately spherical symmetry.
- The apical dendrites of pyramidal neurons can be ignored for the purposes of intra-cortical connectivity (Braitenberg and Schüz 1991).
- Only axo-dendritic connections need be accounted for.
- The volume of dendritic and axonal branching is small compared to the volume in which they distribute themselves and therefore the probability of overlap between the respective trees of any two given cells is small.
- Vertical and horizontal anisotropies in cortical organization can be ignored in the first approximation.
- Dendritic fibre density, based on the dendrograms of Sholl (1953), is described by \( a \exp\left[\frac{-r}{r_0}\right] \), where \( r \) is the radial distance from the cell body.
- Axonal fibre density can be described by a radially homogeneous exponential distribution of the form \( a_{ax} \exp\left[-r/r_a\right] \), where \( r_a \) is the axonal space constant, and \( a_{ax} \) is the ‘density’ of the axonal tree at \( r = 0 \).

The method uses established quantitative anatomical measurements in the calculations. These are given in table 3. The expected number of synapses between a presynaptic cell \( i \) and a postsynaptic cell \( j \) has been derived, using the assumptions described, as

\[
N_{ij}(s) = \frac{\pi \epsilon}{2} \int_0^\infty A_i(r)D_j(r - s) \, d^3r
\]

where \( \epsilon \) is the combined mean radii of axon and dendrite, \( A(r) \) and \( D(r) \) are the axonal and dendritic fibre length densities, respectively, and \( |s| \) is the distance separating the pre- and postsynaptic cells. This can be analytically evaluated to yield

\[
N_{ij}(r) = \pi^2(r_d + r_{ax})a_{ax} Q_{ij}(r)
\]

with

\[
Q_{ij}(r) = \frac{4}{r} \left( \frac{r_a r_0}{r_d^2 - r_0^2} \right)^3 \left\{ r_a \exp\left[-r/r_a\right] \left(r(r_d^2 - r_0^2) - 4r_a r_0^2 \right) + r_0 \exp\left[-r/r_0 \right] \left(r(r_d^2 - r_0^2) + 4r_a r_0^2 \right) \right\}
\]
Table 3. Summary of empirical data and derived values for connectivity in homotypical murine cortex.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Empirical value</th>
<th>Derived value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{ax}$</td>
<td>radius of an axonal fibre ($\mu$m)</td>
<td>0.15</td>
<td>—</td>
</tr>
<tr>
<td>$r_d$</td>
<td>radius of a dendritic fibre ($\mu$m)</td>
<td>0.45</td>
<td>—</td>
</tr>
<tr>
<td>$\rho$</td>
<td>cell density ($\text{mm}^{-3}$)</td>
<td>$9 \times 10^4$</td>
<td>—</td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>relative pyramidal cell density</td>
<td>0.85</td>
<td>—</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>relative stellate cell density</td>
<td>0.15</td>
<td>—</td>
</tr>
<tr>
<td>$L_{D}^{p}$</td>
<td>stellate cell dendrite length (mm)</td>
<td>2.16</td>
<td>—</td>
</tr>
<tr>
<td>$L_{D}^{e}$</td>
<td>total pyramidal cell dendrite length (mm)</td>
<td>3.08</td>
<td>—</td>
</tr>
<tr>
<td>$L_{A}^{p}$</td>
<td>stellate cell axon length (mm)</td>
<td>20</td>
<td>—</td>
</tr>
<tr>
<td>$L_{A}^{e}$</td>
<td>total local pyramidal axonal length (mm)</td>
<td>20</td>
<td>—</td>
</tr>
<tr>
<td>$a_{ax},r_{ax}$</td>
<td>parameters describing branching of both pyramidal basal and stellate dendrites ($\mu$m$^{-2}$, $\mu$m)</td>
<td>0.0028, 31.25</td>
<td>—</td>
</tr>
<tr>
<td>$a_{ax},r_{d}$</td>
<td>parameters describing branching of the local pyramidal cell axon ($\mu$m$^{-2}$, $\mu$m)</td>
<td>—</td>
<td>$7.95 \times 10^{-4}$, 99.1</td>
</tr>
<tr>
<td>$d_{A}^{a}$</td>
<td>stellate axonal intersynapse spacing ($\mu$m)</td>
<td>1.25–5.0</td>
<td>4.45</td>
</tr>
<tr>
<td>$d_{A}^{p}$</td>
<td>pyramidal axonal intersynapse spacing ($\mu$m)</td>
<td>1.25–5.0</td>
<td>6.81</td>
</tr>
<tr>
<td>$d_{D}^{a}$</td>
<td>stellate dendritic intersynapse spacing ($\mu$m)</td>
<td>0.33</td>
<td>0.68</td>
</tr>
<tr>
<td>$d_{D}^{p}$</td>
<td>pyramidal dendritic intersynapse spacing ($\mu$m)</td>
<td>0.4</td>
<td>0.68</td>
</tr>
<tr>
<td>$N_{D}^{total,i}$</td>
<td>total number of synapses per stellate cell dendrite</td>
<td>6545</td>
<td>3570</td>
</tr>
<tr>
<td>$N_{D}^{total,e}$</td>
<td>total number of synapses per pyramidal cell basal dendrite</td>
<td>6545</td>
<td>3570</td>
</tr>
<tr>
<td>$N_{A}^{total,i}$</td>
<td>total number of synapses per stellate cell axon</td>
<td>4000–16000</td>
<td>3570</td>
</tr>
<tr>
<td>$N_{A}^{total,e}$</td>
<td>total number of synapses per pyramidal cell local axon</td>
<td>4000–16000</td>
<td>3570</td>
</tr>
</tbody>
</table>

where $N_{ij}$ is the expected number of synaptic contacts and $r$ is the inter-cellular separation. It should be noted that in general $N_{ij} \neq N_{ji}$. For further details, see Liley and Wright (1994, 1995).

Given that the probability that a given segment of axon selected at random will intersect a dendrite somewhere in the region of co-arborization is small and given that the number of such axonal segments will probably be large, the distribution of connections is expected to be reasonably well described by a Poisson distribution

$$p(q, r) = \frac{\exp[-N_{ij}(r)] N_{ij}(r)^q}{q!}$$

where $p$ is the probability that a postsynaptic neuron will receive $q$ connections from a presynaptic neuron, whose cell bodies are separated by a distance $r$. Equation (5) thus forms the basis for defining a systematic connective asymmetry between model neurons that is anatomically veracious. Such a detailed formulation may seem unnecessary given the considerable variations in intra-cortical connectivity and the associated experimental estimates that are likely to exist both inter-areally and between brains. However, the results of this analysis should be seen as motivating plausible values of connective asymmetry, characteristic scales of connectivity and anatomical connection strengths such that the effects of systematic perturbations of these parameters on network dynamics may be observed. Further, such a formulation is capable of providing an indication of the relationship between inter-cellular separation and the dendritic location of synapse formation (Liley and Wright 1994).
To deal with effects of finite network size, toroidal and reflective boundary conditions were used. Toroidal bounds were implemented by connecting opposite edges of the network array to each other. This condition was used in networks of up to 6400 neurons. Reflecting connections were implemented, in networks of up to 1600 neurons, in the following manner: connections that would have otherwise formed with neurons outside the array were connected to neurons with reflected Euclidean coordinates inside the array. The number of synapses forming between model neurons was calculated using equations (3) and (4), on the basis of 15% inhibitory and 85% excitatory cells (Braitenberg and Schüz 1991). This gave rise to a systematic asymmetry between neurons that depended on inter-cellular separation. Because the number of synapses each of our model neurons received was less than would be expected in real cortical neurons, the weight of each model synapse was modified accordingly. For example, a model excitatory neurons received on average 100 connections from other neurons in the network.

4. Simulation and data analysis

Networks were simulated using the GENESIS neural simulation software (Bower and Beeman 1995) running on a 12 CPU SGI PowerChallenge. The maximum number of cells simulated was 6400 and at this scale open (absorbing or zero) boundary conditions were also applied. The simulation time step was 0.05 ms; however, smaller time steps were tried to assure numerical convergence and stability. Each simulation was run for between 1 and 10 s. Initial conditions were always set to an unperturbed resting state. Input (excitatory synaptic activation) was provided by either independent uncorrelated Poisson pulse trains to approximately 1% and 10% of all network excitatory synapses or by a homogeneously applied constant (time-invariant) current to the soma of all excitatory neurons. The values (mean rates and current) were chosen as being representative of the magnitude of specific and non-specific cortical input (Nunez 1981, 1995) but could also represent the contributions of endogenous network noise due to spontaneous neurotransmitter release or spontaneous neuronal firing.

The soma membrane potential and the occurrence of a spike was recorded for each neuron in the network for every simulation time step. Population averages for both spike occurrence and soma membrane potential were calculated for both excitatory and inhibitory neurons. Population oscillatory frequencies were characterized by power spectral estimates of the mean soma membrane potential for the respective neuronal sub-populations.

5. Simulated wave–pulse relationships

As outlined in section 1, we are interested in the relationship between neural activity at the microscopic (cellular) level and the activity of aggregates of neurons at the macroscopic (columnar) level. To this end we need one or more empirical indices that allow us to bridge events at these different scales. The mean soma membrane potential (‘wave’ in Freeman’s terminology) of the aggregate and the pulse trains of the neurons in this aggregate are such indices, as they will be manifestations of the active state of the neural aggregate. By determining a relationship between the mean soma membrane potential and the probability of neuronal firing, a functional relationship between two scales of description will have been achieved. A convenient way of deriving a relationship between mean soma membrane potential and pulse probability, which makes no assumption about the form of such a relationship, is by calculating a normalized conditional probability density (NCPD) surface (Freeman 1975). In effect this
procedure gives the probability of a neuron firing conditional upon the binned value of the z-normalized mean soma membrane potential and binned time lag. Cross sections of this surface for a fixed time lag define a pulse probability sigmoid curve (PPSC), whereas cross sections for a fixed value of the mean soma membrane potential define a pulse probability wave (PPW). Details of the calculation of the NCPD, PPW and the PPSC can be found in Freeman (1975, 1979).

6. Results

In order to assess model plausibility a wide range of physiological parameters were systematically varied. The corresponding model parameters varied included: (i) stimulus (driving) intensity (equivalent fraction of excitatory synapses activated, magnitude of current, mean spike input rate), (ii) boundary conditions (toroidal, open, reflective), network size ($N = 900, 1600, 3600$ and $6400$), (iii) time scales of fast excitatory and inhibitory channel kinetics, and (iv) relative refractory period durations. The exploration of the resulting parameter space yielded five qualitatively distinct dynamical regimes. All dynamical regimes were accessible solely by variations in the mean input rate or by variations in the magnitude of the homogeneously applied time-invariant current. Qualitatively the dynamical regimes identified were (in ascending order of stimulus intensity required to elicit them):

Regime I: Zero firing, non-oscillatory state.
Regime II: Very low firing rate, non-oscillatory state.
Regime III: Low firing rate, oscillatory state.
Regime IV: High firing rate, oscillatory state.
Regime V: Very high firing rate, non-oscillatory state.

Systematic exploration of the parameter space revealed no other qualitatively discernable regimes. The dynamical regimes III and IV were distinguished by differences in the distribution of the mean excitatory soma membrane potential power spectral density and in the associated

![Figure 2](image_url)

**Figure 2.** Power spectral density of the mean soma membrane potential for excitatory cells for (a) dynamical regime III for a network driven with spike inputs at 1200 spikes/s to 100% of cells, and (b) dynamical regime IV for a network driven with spike inputs at 2400 spikes/s to 100% of cells. The network in (b) shows regime IV type behaviour. Both networks were comprised of 900 neurons and toroidal boundary conditions were used. All other values are as given in the text.
Figure 3. (a) Interspike interval histogram of excitatory cells for dynamical regime III. The interspike intervalgram of a network in regime III, but driven by current homogeneous current input, is visually identical. (b) Standard deviation of spike interval versus mean spike interval for dynamical regime III. The near unity (slope = 0.925) linear relationship shows that the distribution of spike intervals was Poisson. For both plots time units are given in computational time steps of 0.05 ms. Neurons were driven by spike input at 10 different rates between 750 and 1200 spikes/s. Each input epoch was run for 1.25 s and the first 0.25 s were discarded. Data were excluded if the number of spikes generated was less than five. All other parameters are as for figure 2(a).

single-neuron firing statistics. Neither dynamical regime was significantly influenced by changes in the network boundary conditions or network size. Phase differences between the mean soma membrane potentials of excitatory and inhibitory neural populations were negligible. Regime III had relatively low mean firing rates (excitatory population 10–15 pulses/s, inhibitory population 50–60 pulses/s), whereas regime IV was characterized by substantially higher firing rates (excitatory population ~100 pulses/s, inhibitory population ~180 pulses/s). The power spectral density for regime III was broadly distributed between 0 and 30 Hz, whereas regime IV had power sharply concentrated in the range 10–20 Hz. Typical power spectra for these two regimes are illustrated in figure 2.

The two oscillatory domains, III and IV, were also readily distinguished by inter-spike interval (ISI) histograms for excitatory and inhibitory cells. Regime III exhibited Poisson firing statistics as evidenced by a near-linear relationship between the mean and the standard deviation of the ISI. Figure 3 shows an ISI histogram for a randomly chosen excitatory cell. Such an exponentially distributed ISI was present regardless of whether the network was stimulated (driven) with stochastic (Poisson) spike trains or homogeneously applied current. This indicated that the ISI distribution was not a consequence of stochasticity in the input. In contrast, the ISI distribution for the high-firing oscillatory domain IV was not Poisson. Instead neurons in the network exhibited bursting behaviour dependent on the phase of the dominant frequency of the population oscillation.

Another difference in the qualitative behaviour of dynamical regimes III and IV becomes apparent when the spatial properties of network dynamics are investigated. For instance, inputs confined to a central square patch of excitatory and inhibitory neurons are capable of inducing propagating oscillatory activity. For the low-firing rate oscillatory regime III the amplitude of oscillatory activity falls away sharply the further removed from the site of stimulation.
Figure 4. The spatio-temporal activity of a square array of 6400 simulated neurons in response to a random (excitatory Poisson spike train input, mean interspike interval 10 ms) 5% of the excitatory cells. Boundary conditions were absorptive. Successive images represent frames of network activity at approximately 10 ms intervals. The soma membrane potential is represented as both height and colour, with the colour scale varying from hyperpolarized (blue) to depolarized (red).

In contrast, the high-firing rate oscillatory regime IV shows undamped travelling waves of activity which continue to the edges of the network. Such wave-like activity persists even if a random distributed fraction of excitatory neurons is stimulated with Poisson spike trains. Four contiguous snapshots of such activity are illustrated in figure 4. Despite the distributed nature of such input propagating activity consistently emerges from one area (in figure 4 this is the left-hand corner). Different network instantiations gave rise to a different site on the network from where travelling wave activity emerged.

Figure 5 shows derived PPSCs for the dynamical regimes III and IV. These derived curves give the probability of excitatory and inhibitory neurons firing conditional on the mean soma membrane potential of excitatory neurons. They are near-linear along most of their length. The average gradient for the low-firing regime is less that the average gradient for the high-firing regime. Further, the gradient for excitatory cells is consistently greater than that for inhibitory cells.

7. Discussion

The significant finding resulting from the model simulation was the appearance of two distinct oscillatory regimes in response to variations in the magnitude of excitatory input to the network:
a low-firing oscillatory regime and a high-firing oscillatory regime. Similar properties were observed in a one-dimensional continuum model of electrocortical activity sharing a similar parameterization with the present work (Liley et al 1999). Thus we believe that the properties observed do not depend upon discretization of the neuronal elements, nor on limitations of the size of the network able to be simulated.

The low-firing regime was notable in that the mean soma membrane potential was not significantly correlated with the spike trains of individual excitatory or inhibitory neurons. In contrast, in the high-firing domain individual neuronal spike trains were tightly coupled to the phase of the dominant frequency of excitatory neuronal population oscillation. The two oscillatory domains were also distinguished by differences in the distribution of the spectral power for the mean excitatory soma membrane potential.

Variations in transmitter time constants elicited no qualitatively new domains of dynamics, but did alter the distribution of spectral power for the mean excitatory soma membrane potential. For instance, reductions in the time constants associated with fast excitatory and inhibitory channel kinetics that are more consistent with patch clamp data ($\tau_{\text{excitatory}}^1 = 0.5$ ms, $\tau_{\text{excitatory}}^2 = 2.4$ ms, $\tau_{\text{inhibitory}}^1 = 1$ ms and $\tau_{\text{inhibitory}}^2 = 7$ ms) (Lumer et al 1997) gave rise to either a zero-firing non-oscillatory state or a high-firing non-oscillatory state. This is to some extent contrary to the findings of Lumer et al (1997) in which such short channel kinetic time constants gave rise to sustained population oscillations between 40 and 60 Hz. However, this model included slow excitatory (NMDA) and inhibitory (GABAB) neurotransmitter kinetics and model thalamocortical feedback which complicates such a simple comparison. However, Traub et al (1987a, b), in a computational model of rhythmic slow activity in the hippocampus, predicted that the blockade of slow inhibitory postsynaptic potentials would result in the absence of synchronized population oscillations. However, as evidenced by the two oscillatory dynamical regimes (III and IV) found in the simulations performed herein, slow inhibition is not a necessary condition for the emergence of synchronized population oscillations.

One possible effect of the inclusion of slow inhibition in our model might be to determine transitions between low-firing oscillatory dynamics and high-firing oscillatory dynamics by providing a variable bias to pyramidal neurons. In homotypical neocortex such slow

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**Figure 5.** Pulse probability sigmoid curve for excitatory (○) and inhibitory (■) populations in (a) dynamical regime III and (b) dynamical regime IV. In both cases the curves are drawn from their respective z-normalized conditional pulse probability densities at time lag zero. Parameter settings are the same as for figure 1(a).
inhibition might be provided by GABA_B mediated feedback inhibition, perhaps from Martinotti cells (Braitenberg and Schüz 1991) whose axons probably contribute to the surface parallel conduction system in neocortical layer I. GABA_B mediated inhibition has a time course of action between 100 and 200 ms, much longer than the integration times of either local inhibitory or excitatory cells. The sharp transition from low-firing oscillatory dynamics to high-firing oscillatory dynamics is analogous to a phase change in physical systems. This property is reminiscent of Skarda and Freeman’s (1985) findings in the olfactory bulb, where sniffing and resting conditions show contrasting dynamics. Alternation between high- and low-firing rate states may enable single-cell events to interact at a more global scale with EEG phenomena, providing a mechanism by which the different scales can interact (Wright and Liley 1996).

Low-firing rate oscillatory dynamics were notable for the appearance of exponentially distributed ISIs. Because such ISI variability occurred when the network was driven with a homogeneously applied constant current to excitatory cells, it was not a consequence of the statistical properties of the driving stimulus. Such ISI irregularity is at variance with the results of simulations involving the random arrival of excitatory postsynaptic potentials on the spike trains emanating from simple integrate-and-fire neurons (Softky and Koch 1993). In such a model exponentially distributed ISIs could only be obtained if very short, unphysiological, integration time constants (membrane time constant, transmitter-activated channel time constants) were used. It is important to realize that such results were obtained using only a single model neuron randomly driven. Thus the tentative conclusion that can be drawn from the results of the simulation presented here is that the appearance of exponentially distributed ISIs is a consequence of the global dynamics of the networks with the statistical properties of the neuronal firing patterns being explicable as an emergent property. The implication of this conclusion is that microscopic and macroscopic behaviour are constitutive of each other. Such an interpretation is consistent with notions of trans-scale slaving, in which a macroscopic variable (e.g. the mean soma membrane potential or local field potential) constrains, but does not fully determine, microscopic behaviour (e.g. ISI distribution) (Haken 1983). Such trans-scale slaving is believed to be crucial in understanding self-organization in a variety of physical and biological systems.

Alexander and Globus (1996) have described how shifts in dynamical activity can result in cascades of changes up and down the scale hierarchy of the brain. Even subtle changes in the firing patterns of individual neurons can act to change the ‘initial conditions’ leading to the creation of local regions of increased excitation. These local areas must then be capable of perturbing the global brain state by forming the ‘initial conditions’ to attractor formation by an analogous set of mechanisms. This implies that, under appropriate dynamical conditions, an exquisite sensitivity to sensory and recurrent inputs exists in the formation of global EEG defined brain states. Highly erratic perturbation about a state approaching equipartition of energy among resonant modes is apparent in activated EEG (Wright et al 1990), and is consistent with this notion.

In this paper a pseudo-empirical measure has been used to relate properties at one scale with properties at a spatially and temporally subordinate scale, namely the PPSC (pulse probability sigmoid curve). By generating such a curve, a relationship between a macroscopic variable (mean soma membrane potential) and a microscopic variable (probability of single neuronal firing) can be tentatively explored. Based on the requirements of firing rate boundedness and monotonicity the general form for such a relationship is generally assumed to be a sigmoidal function of the mean soma membrane potential or local field potential. Such a sigmoidal nonlinearity is related to the activation or squash function used in the domain of artificial neural networks and is the major nonlinearity present in many continuum field theories purporting to describe bulk electro-rhythmogenenic activity in cortex (e.g. Wilson and Cowan...
1972, 1973, Nunez 1981, 1995, Liley 1997, Robinson et al 1997, Liley et al 1999). The sigmoidal nonlinearity is frequently removed by linearization to obtain analytical solutions in continuum field theories. The near-linear form of the PPSC obtained from our spatially discrete computational model suggests that such linearizations may be considered legitimate in terms of the functional implications of such manipulations. This near-linear form is surprising given that both excitatory and inhibitory neural populations had thresholds drawn from a Gaussian distribution.

It remains to be asked whether the alpha-like activity which emerged in the simulation can be regarded as a source for some of the alpha activity observed in real brains. If our formulation has captured conditions present even transiently, and in a minority of cortical cells, then this mechanism may account for a part at least, of the generation of alpha. A confusing point is that the simulation exhibits close coupling of action potential density with local field potential in the alpha range, and this is not observed in physiological conditions for any EEG rhythm other than gamma-band activity in restricted conditions (Stryker 1989). However, as noted above, the transient generation of rhythmic alpha at restricted locales of excited cortex may be transferred by passive spread to less excited cortex, thus accounting for the decoupling of pulse and wave usually observed in physiological alpha.

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